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THEORY OF SMALL-DIAMETER AIRLIFT PUMPS

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Abstract—The results and discussion of an experimental study of the effects of tube diameter on vertical slug flow, specifically as it relates to 3–25 mm airlift pump performance are presented. The theory previously presented by Nicklin [*Trans. Instn chem. Engrs* **41**, 29–39 (1963)] is extended into this range of tube diameters by taking into account the effects of surface tension on the bubble rise velocity. Differences are noted between the rise velocity of a single gas slug and a train of gas slugs in small vertical tubes. Comparisons are made between experimental observations and theoretical predictions. Good agreement is observed for Re > 500 and for surface tension numbers between 0.02 and 0.2.

Key Words: airlift pumps, slug flow

INTRODUCTION

A typical airlift pump configuration is illustrated in figure 1. A gas, usually air, is injected at the base of a submerged riser tube. As a result of the gas bubbles suspended in the fluid, the average density of the two-phase mixture in the tube is less than that of the surrounding fluid. The resulting buoyant force causes a pumping action.

The slug flow regime is most widely encountered in airlift pump operation and is characterized by bubbles large enough to nearly span the riser tube. The length of the bubbles ranges from roughly the diameter of the tube, to several times this value. The space between the bubbles is mostly liquid filled and is referred to as a liquid slug (Govier & Aziz 1972). The large gas bubble is also referred to as a gas slug or Taylor bubble.

Extensive experimental and theoretical work has been done on airlift pumps in the slug flow regime (Apazidis 1985; Clark & Dabolt 1986; Hjalmars 1973; Higson 1960; Husain & Spedding 1976; Jeelani *et al.* 1979; Nicklin 1963; Richardson & Higson 1962; Sekoguchi *et al.* 1981; Slotboom 1957; Stenning & Martin 1968). These studies have been confined to air/water systems in tubes with dia > 20 mm in which the effects of surface tension are small and have therefore been neglected.

As tube diameter is decreased below 20 mm, the effects of surface tension on the dynamics of vertical slug flow become increasingly important (Bendiksen 1985; Nickens & Yannitell 1987; Tung & Parlange 1976; White & Beardmore 1962; Zukoski 1966). It has been speculated that increased efficiency might be obtained by using small-diameter tubes at low flow rates (Nicklin 1963). Neither a satisfactory theory, not conclusive experimental evidence, however, has as yet been presented for small-diameter airlift operation. The objective of this study is to examine the effects of tube diameter on the hydrodynamics of the airlift pump in the range of tube diameters in which surface tension effects are significant.

THEORY

In previous work, the rise velocity of a gas slug in a vertical tube relative to a moving liquid slug has been described by (Bendiksen 1985; Collins *et al.* 1978; Griffith & Wallis 1961; Nicklin *et al.* 1962):

$$V_T = C_0 V_m + V_{Ts}, \qquad [1]$$

where

V_T = rise velocity of the Taylor bubble (m/s),

V_{Ts} = rise velocity of the Taylor bubble in still fluid (m/s),

C_0 = liquid slug velocity profile coefficient

and

V_m = mean velocity of the liquid slug (m/s), given by

$$V_m = \frac{Q_L + Q_G}{A}, \qquad [2]$$

where

Q_L = volumetric liquid flow rate (m^3/s),

Q_G = volumetric gas flow rate (m^3/s),

and

A = tube cross-sectional area (m^2).

Following the analysis used by Nicklin (1963), the velocity of the Taylor bubble is set equal to the average linear velocity of the gas in the riser tube:

$$V_T = \frac{Q_G}{\epsilon A}, \qquad [3]$$

where

ϵ = gas void ratio.

It is convenient to express the volumetric liquid and gas flows and bubble velocity in dimensionaless form as Froude numbers, defined by:

$$Q'_L = \frac{Q_L}{A(gD)^{1/2}}; \quad Q'_G = \frac{Q_G}{A(gD)^{1/2}}; \quad V'_{Ts} = \frac{V_{Ts}}{(gD)^{1/2}}; \qquad [4]$$

where

Q'_L = dimensionless volumetric liquid flow,

Q'_G = dimensionless volumetric gas flow,

V'_{Ts} = dimensionless bubble rise velocity in still fluid,

D = tube diameter (m),

and

g = acceleration due to gravity (m/s^2).

From [1]–[4], the gas void fraction in the riser tube can be expressed as

$$\epsilon = \frac{Q'_G}{C_0(Q'_L + Q'_G) + V'_{Ts}}. \qquad [5]$$

The submergence ratio is a parameter commonly found in airlift analysis and is defined as

$$\alpha = \frac{Z_s}{Z_l + Z_s}, \qquad [6]$$

where

α = submergence ratio,

Z_l = lift height (m) (see figure 1),

and

Z_s = length of tube submerged (m).

The submergence ratio is equal to the average pressure gradient along the riser tube which is made up of components due to the weight of the two-phase mixture and frictional losses. Performing a static pressure balance on a vertical tube which is submerged in fluid (see figure 1), it follows that:

$$\rho g Z_s = \rho g(1-\epsilon)(Z_s + Z_l), \qquad [7]$$

where

$$\rho = \text{fluid density (kg/m}^3\text{)}.$$

This assumes that the weight of the gas is negligible relative to the weight of the liquid. If the fluid in the tube is moving, an additional pressure drop due to frictional losses must be added to the r.h.s. of [7]. The single-phase frictional pressure drop can be calculated based upon the mean slug velocity as

$$P_s = f\frac{(Z_s + Z_l)\rho V_m^2}{2D}, \qquad [8]$$

where

$$P_s = \text{single-phase frictional pressure drop (N/m}^2\text{)},$$

$$f = \text{friction factor (Giles 1962)}$$

$$= \frac{0.316}{\mathrm{Re}^{0.25}}, \qquad [9]$$

$$\mathrm{Re} = \frac{DV_m}{\nu} \qquad [10]$$

and

$$\nu = \text{kinematic fluid viscosity (m}^2\text{/s)}.$$

The single-phase frictional loss must then be multiplied by $(1-\epsilon)$, the fraction of the tube occupied by the liquid slugs, to obtain the total frictional pressure drop in the riser tube. The frictional effects in the liquid film around the gas bubble have been shown to be small compared to those in the liquid slug and are therefore neglected (Nakoryakov *et al.* 1986).

Including the frictional effects in the pressure balance results in

$$\rho g Z_s = \rho g(1-\epsilon)(Z_s + Z_l) + f\frac{(Z_s + Z_l)\rho V_m^2}{2D}(1-\epsilon). \qquad [11]$$

Dividing both sides by $[\rho g(Z_s + Z_l)]$ and rearranging gives

$$\alpha = (1-\epsilon)\left[1 + \frac{f}{2}(Q'_L + Q'_G)^2\right]. \qquad [12]$$

Thus, for a given tube diameter, imposing the gas flow rate and the submergence ratio, the liquid flow rate may be determined using the system of equations summarized in table 1.

Table 1. Summary of the airlift equations

$\epsilon = \dfrac{Q'_G}{1.2(Q'_L + Q'_G) + V'_{Ts}}$	$\alpha = (1-\epsilon)\left[1 + \dfrac{f}{2}(Q'_L + Q'_G)^2\right]$	
$Q'_L = \dfrac{Q_L}{A(gD)^{1/2}}$	$Q'_G = \dfrac{Q_G}{A(gD)^{1/2}}$	$\alpha = \dfrac{Z_s}{Z_l + Z_s}$
$V'_{Ts} = 0.352(1 - 3.18\,\Sigma - 14.77\,\Sigma^2)$		
$f = \dfrac{0.316}{\mathrm{Re}^{0.25}}$	$\mathrm{Re} = \dfrac{D(Q_L + Q_G)}{\nu A}$	$\Sigma = \dfrac{\sigma}{\rho g D^2}$

It is usual to define the efficiency of the airlift pump as the net work done in lifting the liquid, divided by the work done by the isothermal expansion of the air (Nicklin 1963):

$$n = \frac{Q_L Z_l \rho g}{Q_G P_a \ln\left(\frac{P_0}{P_a}\right)}, \qquad [13]$$

where

n = efficiency,

P_a = atmospheric pressure (N/m^2),

and

P_0 = pressure at base of riser tube (N/m^2).

Nicklin (1963) introduced the concept of point efficiency which is accurate in describing total airlift efficiency to within 1% for submergence lengths of up to 5 m:

$$n = \frac{Q'_L(1-\alpha)}{Q'_G \alpha}. \qquad [14]$$

Two important effects become significant when the airlift tube diameter is below about 20 mm. The first is increased importance of surface tension. The second is decreased Re. The effects of surface tension can be characterized by the inverse Eötvos number or surface tension number, Σ, defined as

$$\Sigma = \frac{\sigma}{\rho g D^2}, \qquad [15]$$

where

Σ = surface tension number

and

σ = surface tension (N/m).

White & Beardmore (1962) have defined a dimensionless parameter which relates only to the properties of the fluid and expresses the relative importance of viscosity to surface tension:

$$Y = \frac{g\mu^4}{\rho\sigma^3}, \qquad [16]$$

where

Y = fluid viscosity/surface tension parameter

and

μ = fluid viscosity (kg/m s).

Experimental results have shown that when this parameter is $< 10^{-8}$ (which is the case for an air/water system), the viscosity does not influence the bubble rise velocity in still fluid (White & Beardmore 1962).

Theoretical and experimental analyses of the rise velocity of a single gas slug in still fluid have shown that when both surface tension and viscous effects are negligible, the bubble Froude number in still fluid (B) assumes a constant value of about 0.35 (Bendiksen 1984; Collins *et al.* 1978; Davies & Taylor 1950; Higson 1960; Nakoryakov *et al.* 1986; Nickens & Yannitell 1987; Nicklin *et al.* 1962; White & Beardmore 1962; Zukoski 1966). This is the value which has been used in previous airlift analysis (Nicklin 1963; Clark & Dabolt 1986).

The value of B is influenced by surface tension when the surface tension parameter is above about 0.02 (Bendiksen 1984, 1985; Nickens & Yannitell 1987; Tung & Parlange 1976; Zukoski 1966). This corresponds to a tube diameter less than about 20 mm in an air/water system. As the tube diameter is decreased below this value, B decreases. When the surface tension number is above about 0.2 the bubble will not rise in still fluid and the value of B is zero. This corresponds to a tube diameter of about 6 mm in an air/water system. When the effects of viscosity can be neglected, as is the case for an air/water system, B can be expressed as a function of the surface tension parameter alone (Nickens & Yannitell 1987; White & Beardmore 1962):

$$V'_{Ts} = 0.352\ (1 - 3.18\,\Sigma - 14.77\,\Sigma^2). \qquad [17]$$

Correction can also be made on B for other gas/liquid systems when viscous effects are significant (Nickens & Yannitell 1987; White & Beardmore 1962).

Theoretical analyses of bubble rise velocity have applied potential flow theory at the bubble tip, expressing the stream function of the flow in terms of a Bessel function series of the first kind and first order (Bendiksen 1985; Nickens & Yannitell 1987; Tung & Parlange 1976). This treatment of the hydrodynamics only at the bubble tip has been justified by several experimental studies in which air/water bubble dynamics have been shown to be independent of bubble length (Nicklin *et al.* 1962; Griffith & Wallis 1961). The effects of surface tension are accounted for in the application of the boundary condition of constant gas pressure along the bubble surface. As the radius of curvature of the bubble is reduced, surface tension acts to increase the pressure at the gas/liquid interface. This changes the flow dynamics at the bubble surface and hence the bubble rise velocity.

Nicklin *et al.* (1962), have shown that a value of 1.2 for C_0 is suitable when the liquid slug Re is >8000. For airlift pumps with dia >20 mm, the Re is usually >8000. The Re can be considerably less than 8000 for airlift pumps with dia <20 mm, however.

An increase in the velocity profile coefficient has been observed for Re <8000 (Bendiksen 1985; Nicklin *et al.* 1962). The limiting value of the velocity profile coefficient has been found to be about 2 for Re values approaching zero. This rise in C_0 has also been predicted theoretically when a laminar velocity profile was imposed in the liquid ahead of the gas slug (Bendiksen 1985; Collins *et al.* 1978).

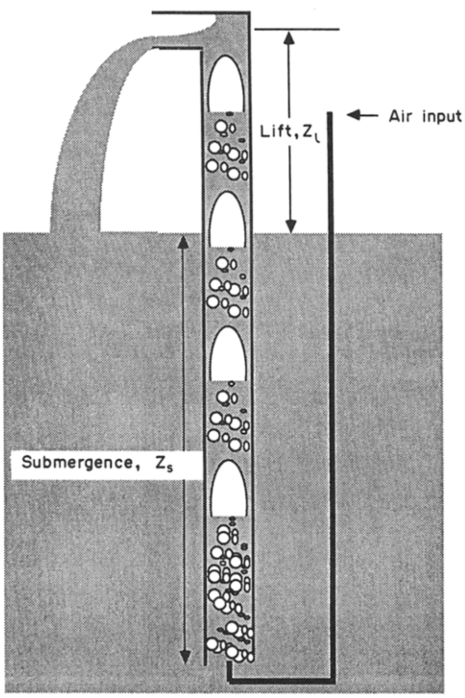

Figure 1. Typical airlift pump.

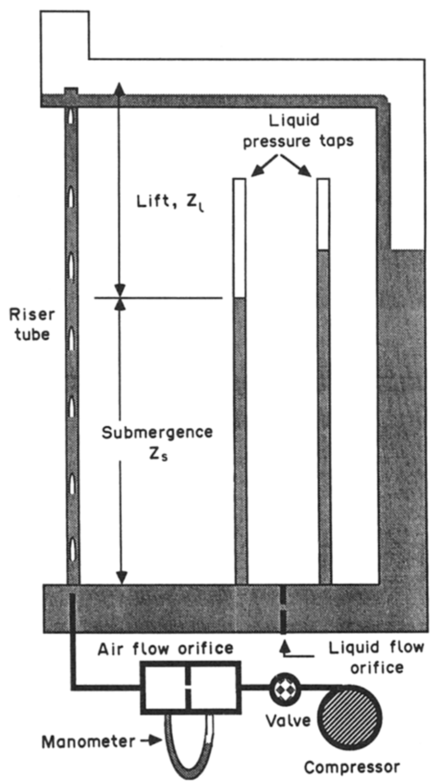

Figure 2. Experimental apparatus.

The bubble rise velocity, as expressed in [1], can thus be interpreted as its rise velocity in still fluid plus the velocity of the fluid encountered at its tip. The velocity profile coefficient is then the ratio of the liquid velocity at the tube axis to the average velocity of the liquid slug. The limiting values of C_0 (1.2 for high Re and 2.0 for low Re reflect either turbulent or laminar velocity profiles in the liquid slug.

Neglecting frictional effects, the efficiency of the airlift from [5], [10] and [12] is:

$$n = \frac{Q'_L}{C_0(Q'_L + Q'_G) + V'_{Ts} - Q'_G}. \qquad [18]$$

Decreasing tube diameter is the range where surface tension effects are significant will decrease the value of the bubble Froude number, V'_{Ts}. This will increase efficiency. Previous experimental work has shown that a reduction in the liquid slug Re will increase C_0 if the transition to a laminar velocity profile occurs in the liquid slugs. This will reduce efficiency. Thus, two opposing effects are predicted. An experiment was performed to determine the relative importance of the two effects.

EXPERIMENTAL PROCEDURE

The test apparatus is illustrated in figure 2. The reservoir and return sections were glass tubes with a 38 mm i.d. The riser tubes were 1.80 m in length and range in i.d. from 3.18 to 19.1 mm. Volumetric air and water flow rates, bubble rise velocity, submergence and lift height were measured after the flow stabilized for each trial.

Air and water flows were determined by means of pressure drop measurements across calibrated orifices. Bubble rise velocities in both still and moving liquid were determined by timing a bubble over a known travel distance. The flow was allowed to develop for a distance of 0.8 m before bubble velocity measurements were started. Slug flow developed within 1–5 dia of the entrance for all of the riser tubes and flow rates tested.

The static head at the pressure tap immediately before the riser tube was used as a reference level in determining lift height and submergence (see figure 2). This same pressure was used as the air inlet pressure. By using this pressure as a reference, all losses in the water return line, air supply line and across the orifices were separated from the riser tube measurements. The resulting experimentally measured flow variables are therefore as close as possible to measuring the conditions of the riser tube alone.

Submergence ratios were varied by changing the amount of fluid in the reservoir. Air was injected into the system by means of a small diaphragm-type compressor. The air flow rate was controlled by a valve between the compressor and the air flow measurement orifice.

The velocity profile coefficient was determined using [1], and [2] with measured flow rates and bubble rise velocities. The experimental efficiency was determined using [12] with measured values of the liquid flow, gas flow and submergence ratio.

RESULTS AND DISCUSSION

For all of the tube sizes tested, the bubble rise velocity in still fluid corresponded very closely to the prediction equation used and results reported by previous workers (White & Beardmore 1962; Tung & Parlange 1976; Zukoski 1966). Experimental results for tubes with 3.18, 6.35 and 9.53 mm dia, showed the velocity profile coefficient scattered closely about 1.2 with no increasing trend for Re decreasing to as low as 500 (see figure 3). This differs from earlier results in which the velocity profile coefficient increased for Re <8000 (Bendiksen 1984; Nicklin *et al.* 1962). The experiment was repeated using a 19.1 mm dia tube to determine whether surface tension effects influenced this phenomenon. For this tube size surface tension effects were negligible, as in previous studies. The results again showed no increasing trend in the velocity profile coefficient for low Re.

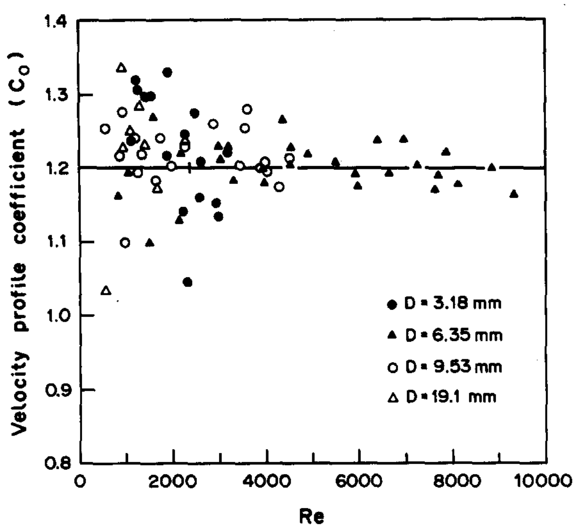

Figure 3. Velocity profile coefficient vs Re.

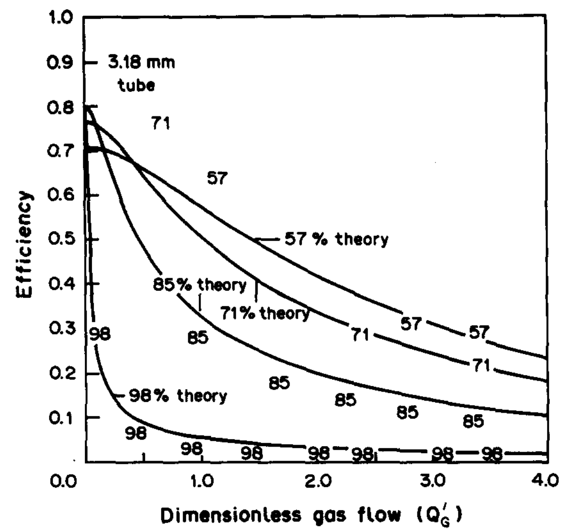

Figure 4. Efficiency vs gas flow, 3.18 mm tube. Numbers on the graph body indicate the percentage submergence of experimental points. Solid lines indicate theoretical lines of constant submergence.

There are two major differences to be noted between the previous experiments (Bendiksen 1984; Nicklin *et al.* 1962) and the present one:

1. In the previous experiments the motion of a single gas slug moving through a moving stream of liquid was studied. In the present experiment the gas was introduced continuously, resulting in a series of gas slugs moving through a series of liquid slugs.
2. The previous experiments used a pump to regulate the liquid flow, whereas in the present experiment, liquid motion was the result solely of buoyancy.

When a single gas slug is placed in a stream of liquid whose motion is pump driven, the velocity profile in the liquid ahead of the gas slug is a result of single-phase pipe flow. When a series of gas and liquid slugs rise concurrently, the velocity profile in the liquid slugs is a result of two-phase slug flow dynamics.

The results of the present experiment show that the liquid slugs have a turbulent velocity profile for Re values as low as 500. Observation of the motion of very small gas bubbles suspended in the liquid slug showed erratic behavior, further confirming the presence of turbulence in the liquid slug at low Re. A laminar velocity profile in the liquid ahead of the gas slug was observed at low Re in previous experiments (Bendiksen 1985; Nicklin *et al.* 1962). It is believed that this difference is the result of the vorticity generated in the liquid film surrounding the gas slugs and in their wake when a series of gas slugs rise concurrently with a series of liquid slugs. A value of 1.2 was used for the velocity profile coefficient in all subsequent theoretical airlift calculations, since a turbulent velocity profile was observed in the liquid slug for the range of flow conditions tested.

The experimentally determined efficiencies vs submergence ratio and gas flow are shown in figures 4–6. Theoretical efficiencies for lines of constant submergence are also shown. The agreement between theory and experiment is good except when the gas flow rate is low and the submergence ratio is <0.7. This region signifies the approach of flow oscillations which are not considered in the theoretical model.

Other workers have observed flow oscillations in large-diameter airlift operation (Apazidis 1985; Higson 1960; Hjalmars 1973; Sekoguchi *et al.* 1981; Wallis & Heasley 1961). Oscillations have been reported to both decrease airlift efficiency (Higson 1960; Richardson & Higson 1962), and increase efficiency (Sekoguchi *et al.* 1981). Measurements taken in the present study, in the region approaching oscillatory behavior, show efficiencies higher than those predicted by theory for the tube sizes tested in this regime.

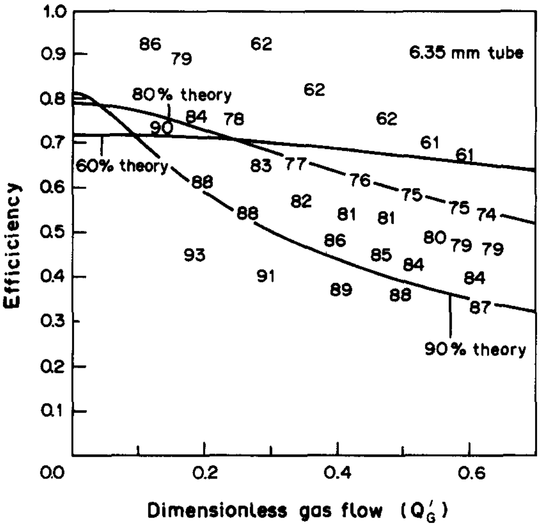

Figure 5. Efficiency vs gas flow, 6.35 mm tube. Numbers on the graph body indicate the percentage submergence of experimental points. Solid lines indicate theoretical lines of constant submergence.

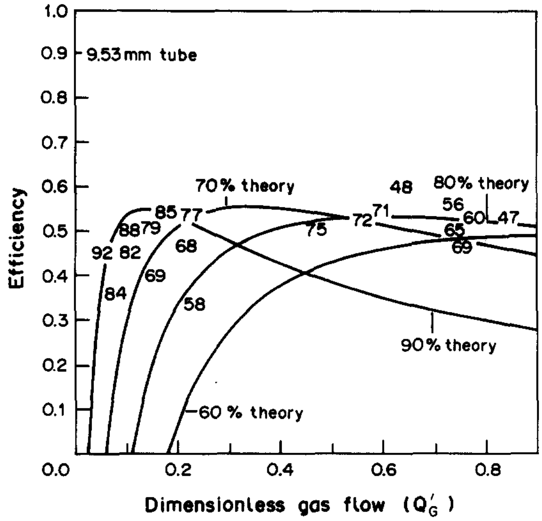

Figure 6. Efficiency vs gas flow, 9.53 mm tube. Numbers on the graph body indicate the percentage submergence of experimental points. Solid lines indicate theoretical lines of constant submergence.

It is instructive to examine the situation in which no frictional losses are included in theoretical predictions. This is an excellent approximation to actual performance at low flow rates when frictional losses are small. Efficiencies will drop increasingly below the frictionless case as flow rates increase (see figure 7). For small tubes (< 6 mm dia) the bubble Froude number is still fluid (V_{Ts}) is zero and the efficiency is constant with respect to gas flow and increases with increasing submergence ratio in the frictionless case.

For large tubes (> 20 mm dia), the bubble Froude number is equal to 0.35, its upper limit, and frictionless efficiency depends on both submergence and flow rate. Negative values of efficiency occur at low flow rates, indicating a situation in which work is done by the expanding gas and no useful work is being performed pumping the fluid.

For tubes in the intermediate size range (6–20 mm), the bubble Froude number falls between its upper and lower limits. Efficiencies fall between the positive values encountered with small tubes and the negative values for large tubes as flow rate decreases.

Frictional losses are negligible at low gas flow rates. Frictional losses increase faster for higher submergence ratios as gas flow increases. This causes the characteristic crossing of the constant submergence ratio efficiency curves (see figure 7).

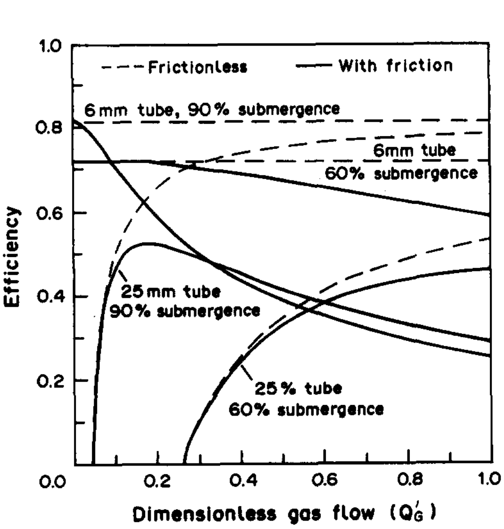

Figure 7. Theoretical efficiency vs gas flow.

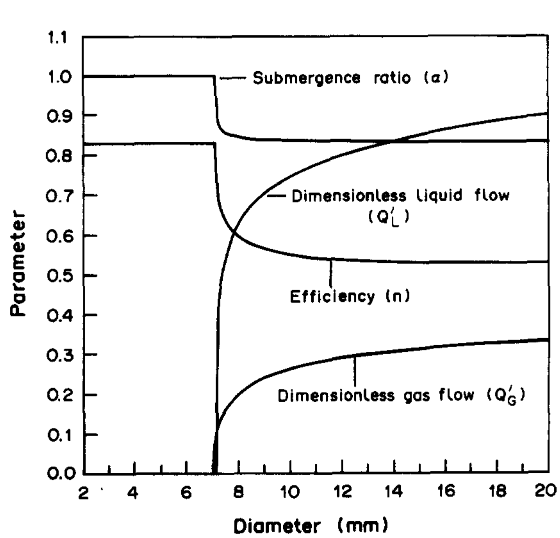

Figure 8. Optimum flow characteristics vs tube diameter.

A summary of the optimal flow characteristics of the airlift pump vs tube diameter is presented in figure 8. Nicklin (1963) concluded that optimal pump efficiency and submergence ratio were insensitive to tube diameter. This is indeed the case for air/water systems when tube diameters are >20 mm and surface tension effects are negligible. As tube diameters are decreased below this value, the effects of surface tension act to increase optimal airlift efficiency and submergence ratio, confirming Nicklin's (1963) speculations. The maximum attainable theoretical airlift efficiency is 83% and occurs for tubes with dia <6 mm in the limit of zero gas flow and 100% submergence.

CONCLUSION

A difference has been observed between single-bubble and bubble-train slug flow in air/water systems at low Re. When a single gas slug rises in a moving liquid stream, the velocity profile coefficient approaches a value of 2.0 for low Re flow in air/water systems. This indicates a laminar velocity profile in the liquid ahead of the gas slug. When a series of gas slugs rise concurrently with a series of liquid slugs, the velocity profile coefficient remains at a value of 1.2 for Re values as low as 500. This indicates turbulent flow in the liquid slugs. It is believed that this difference is the result of the vorticity generated in the liquid film surrounding the gas slugs and in their wake.

It has been shown that including this effect and the effects of surface tension on bubble rise velocity allows the airlift pump theory previously described by Nicklin (1963) to be extended to lower tube diameters in the range 3–20 mm. It has also been shown that airlift efficiency and optimal submergence ratio increase in this range of tube diameters. The theory described here can be used with confidence to design small-diameter airlift pumps.

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